Laboratori Nazionali di Frascati

LNF-64/42 (1964)

M. Ademollo e R. Gatto: WEAK NON-LEPTONIC INTERACTIONS IN THE QUARK MODEL.

Estratto da: Phys. Letters, 10, 339 (1964).

64/42

WEAK NON-LEPTONIC INTERACTIONS IN THE QUARK MODEL

M. ADEMOLLO and R. GATTO
Istituto di Fisica dell'Università, Florence, Italy
Laboratori Nazionali del CNEN, Frascati, Rome, Italy

Received 21 April 1964

In this note we propose a possible way of generating weak non-leptonic interactions in the "quark" model [1, 2] that seems to us rather elegant and possesses attractive features. The resulting weak (strangeness violating) non-leptonic interactions must satisfy $|\Delta S| = 1$ and $|\Delta T| = \frac{1}{2}$. More precisely the weak non-leptonic Lagrangian behaves as an octet with $\Delta T = \frac{1}{2}$ under SU_3.

If all strong interactions are due to the self-coupling of a vector (V) or pseudovector (A) current, symmetrical in the three quarks, as proposed by Gell-Mann [1], the resulting weak non-leptonic interaction must transform - from invariance under CP - as the sixth component of an octet. Gell-Mann [3] has shown that, in the limit of unitary symmetry, such a property quite generally forbids $K_1^0 \to 2\pi$ and gives a relation among the s-wave amplitudes of strange baryon decays.

Strong interaction Lagrangians of the scalar (S), pseudoscalar (P) and tensor (T) types give rise instead to a parity violating weak term that transforms like the seventh component of an octet. Other forms of the strong interaction Lagrangian may also give rise to a parity violating term that transforms like the seventh component of an octet.

The above conclusions on the weak interaction Lagrangian are fully valid in the limit of unitary symmetry. The presence of a symmetry-breaking unsymmetrical mass term does not however affect the derivation of the weak Lagrangian.

Weak non-leptonic interactions are generated in our model from weak perturbations of the free Lagrangian for the quarks. The mass term in the free Lagrangian is not required to be symmetrical. (Addition of an unsymmetrical mass term is a simplest possibility for symmetry-breaking). We write the total Lagrangian as

$$\mathcal{L} = -\bar{t}[\gamma_\mu \partial_\mu (A + b\gamma_5) + (c + \mathrm{i}d\gamma_5)]t - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} + \mathcal{L}_S , \qquad (1)$$

where t describes the triplet of quark fields [1]; A, b, C, and d are hermitian 3×3 matrices in unitary space, $\mathcal{L}_S$ is the strong unitary invariant Lagrangian; $-\frac{1}{4}F_{\mu\nu}F_{\mu\nu}$ is the photon free Lagrangian; and $\partial_\mu = \partial/\partial x_\mu - \mathrm{i}eA_\mu$ where A_μ is the photon field. The matrices A, b, C, and d can have non-zero elements only between states of equal charge. We write $A = 1 + a$, $C = M + m$, where M is diagonal and positive, and assume that the matrices a, b, m, and d are of weak order *. To bring the free quark Lagrangian to the canonical form $-\bar{t}[\gamma_\mu\partial_\mu + M']t$ where M' is diagonal, we transform [4] from the field t to the "physical" field t' writing

$$t = [1 + \tfrac{1}{2}(u_1 + \mathrm{i}u_2) + \tfrac{1}{2}(v_1 + \mathrm{i}v_2)]t' , \qquad (2)$$

where u_1, u_2, v_1 and v_2 are hermitian matrices also of weak order. They must satisfy, to first order in weak terms

$$\begin{aligned} u_1 = -a, \qquad v_1 = -b, \\ \{M, u_1\} + \mathrm{i}[M, u_2] + 2(C - M') = 0 , \\ \{M, v_2\} - \mathrm{i}[M, v_1] + 2d = 0 . \end{aligned} \qquad (3)$$

These equations can in general be solved for a general diagonal positive M and give for the elements of the diagonal matrix M' the corresponding diagonal elements of CA^{-1}. However, if M has the same eigen-

* The positive definiteness of the energy operator requires the condition (satisfied here) that $A + b\gamma_5$ be positive definite, which also assures of the consistency of the anticommutation relations.

value for the two neutral quarks, the matrix equation $M' = CA^{-1}$ must hold, requiring that CA^{-1} be diagonal. The transformation (1) brings the free quark Lagrangian in the canonical form but adds to the strong interaction Lagrangian a weak Lagrangian which is in general non-invariant under SU_3 and contains a parity violating part.

If $\mathcal{L}_s$ has the form of a self-coupling of a unitary singlet vector current

$$\mathcal{L}_s = f(\bar{t}\gamma_\mu t)(\bar{t}\gamma_\mu t) \tag{4}$$

$\mathcal{L}_s$ gets transformed, under (1), into (we omit the dashes)

$$\mathcal{L}_s + f(\bar{t}\gamma_\mu u_1 t)(\bar{t}\gamma_\mu t) + f(\bar{t}\gamma_\mu \gamma_5 v_1 t)(\bar{t}\gamma_\mu t) + \text{h.c.} \tag{5}$$

The hermitian matrices u_1 and v_1 in (5) must be real (to assure of time reversal invariance) and neutrals (no matrix elements between states of different charge). Thus they will be real linear combinations of 1, λ_3, λ_6 and λ_8 in Gell-Mann's notations [6]. The terms proportional to λ_3 and λ_8 lead to weak strangeness-conserving interactions (though parity non-conserving) not easily detectable at present. Non-leptonic decays will thus be described by

$$\mathcal{L}_w = [\bar{t}\gamma_\mu(g + h\gamma_5)\lambda_6 t][\bar{t}\gamma_\mu t] + \text{h.c.}, \tag{6}$$

where g and h are the weak coupling constants. This Lagrangian transforms under SU_3 as the sixth component of an octet with $CP = +1$. It has thus the features suggested by Gell-Mann [3] (in particular it forbids in the unitary limit $K_1^0 \to 2\pi$ and provides Gell-Mann's relation for the s-wave baryon-decay amplitudes). On the other hand relations appealing to a possible invariance under R (unitary reflection) [6] cannot be justified in our model, since the quark model is intrinsically non-invariant under R.

For a pseudovector strong coupling one has only to replace in (5) γ_μ with $\gamma_\mu\gamma_5$. Instead for S, P, T couplings one must replace γ_μ with the relevant covariant and also substitute iv_2 for v_1 in (5): in $\mathcal{L}_w$ the parity violating term would then behave as the seventh component of an octet with $CP = +1$.

Similarly, with

$$\mathcal{L}_s = f(t\gamma_\mu\lambda_i t)(t\gamma_\mu\lambda_i t) \tag{7}$$

the weak Lagrangian for non-leptonic decays becomes

$$\mathcal{L}_w = \tfrac{1}{2}f[(\bar{t}\gamma_\mu\{u_1, \lambda_i\}t) + (\bar{t}\gamma_\mu\gamma_5\{v_1, \lambda_i\}t) - i(\bar{t}\gamma_\mu\gamma_5[v_2, \lambda_i]t)][\bar{t}\gamma_\mu\lambda_i t] + \text{h.c.} \tag{8}$$

Invariance under time reversal requires u_1 and v_1 to be proportional to λ_6 and v_2 proportional to λ_7. Thus, unless $i[C, b] + 2d = 0$ in the original Lagrangian, a parity violating term behaving as the seventh component of an octet with $CP = +1$ would be present in $\mathcal{L}_w$. With use of the tensors [5] f_{ijk} and d_{ijk} one can write (8) as

$$\mathcal{L}_w = d_{6ik}[\bar{t}\gamma_\mu(g+h\gamma_5)\lambda_k t][\bar{t}\gamma_\mu\lambda_i t] + \tfrac{2}{3}[\bar{t}\gamma_\mu(g+h\gamma_5)t][\bar{t}\gamma_\mu\lambda_6 t] + h'f_{7ik}[\bar{t}\gamma_\mu\gamma_5\lambda_k t][\bar{t}\gamma_\mu\lambda_i t] + \text{h.c.} \tag{9}$$

with g, h and h' real. In eq. (9) both unitary symmetric and unitary antisymmetric couplings are present. One can check directly that the parity conserving part commutes with C (charge conjugation), while the parity violating part anticommutes [3]. If $\mathcal{L}_s$ (in eq. 7) is of the A type everything is unchanged apart from the substitution $\gamma_\mu \to \gamma_\mu\gamma_5$. If $\mathcal{L}_s$ is S, P, or T, apart from substitution of the covariants, one must also substitute $v_1 \rightleftharpoons iv_2$. The unitary transformation properties of the two parity violating terms are then interchanged.

Finally we must point out that the above views on the non-leptonic weak interactions do not contain any justification for a possible equality in strength between non-leptonic and leptonic or semileptonic couplings.

1) M. Gell-Mann, Physics Letters 8 (1964) 214.
2) G. Zweig, CERN, preprint.
3) M. Gell-Mann, Phys. Rev. Letters 12 (1964) 155; see also N. Cabibbo, Phys. Rev. Letters 12 (1964) 62.
4) See N. Cabibbo, R. Gatto and C. Zemach, Nuovo Cimento 15 (1960) 168.
5) M. Gell-Mann, Phys. Rev. 125 (1962) 1067.
6) B.W. Lee, Phys. Rev. Letters 12 (1964) 83;
B. Sakita, Phys. Rev. Letters 12 (1964) 379.

* * * * *